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ABSTRACT

The development of proportional reasoning is a major focus of the middle grades curriculum. The challenge for educators is to find contexts that engage students and that facilitate the study of proportional reasoning. This chapter explores proportional thinking with students in grades 3-8 by using a number of books in which the underlying stories center on proportional thinking. Results indicate that literature provides an effective vehicle for engaging middle school students in mathematics. Middle grades students became engaged in doing mathematics as a result of reading books, and the contexts seem to offer insight into ways in which to engage in mathematics. (KHR)

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Using Literature as a Vehicle to Explore Proportional Reasoning

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As indicated in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000), the development of proportional reasoning is a major focus of the middle-grades curriculum. Although much of the formal study of proportions occurs in the middle grades, much informal work with proportional reasoning through multiplicative situations is appropriate in the upper elementary grades. The challenge for educators is to find contexts that engage students and that facilitate the study of proportional reasoning, either formally or informally.

Literature provides a natural context in which this study can occur. A number of authors have focused on the use of literature at the elementary school level to engage students in exploring mathematics (Whitin and Wilde 1992, 1995). Some work is beginning to appear about the use of literature to engage middle grades students in studying mathematics (Austin and Thompson 1997; Lawrence 1999).

A number of literature titles exist in which the underlying story centers on proportional thinking. At the upper elementary grades, these books can be used to study ratios and proportions informally by focusing on the multiplicative relationships inherent in the story. At the middle grades, these same books can be used to study ratios and proportions more formally. We have used a number of these books to explore proportional thinking with students, informally in grades 3–5 and both formally and informally in grades 6–8.

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MULTIPLICATIVE REASONING IN THE ELEMENTARY GRADES

With students in the intermediate grades, we have focused on proportional reasoning not through formal study of ratios and proportions but informally through multiplicative ideas. Consider the book *Jim and the Beanstalk* (Briggs 1970). In this story, Jim is the son of Jack (from Jack and the Beanstalk fame) and the giant is the son of the giant from the earlier story. But the current giant has problems: he cannot read because of eye difficulties, he cannot eat because of teeth problems, and he believes he is ugly because he has no hair. To solve each problem, Jim takes appropriate measurements and has objects made to solve the giant's problems. For instance, to solve the eye problem, Jim measures the giant's head, and with a large coin pays to have a pair of reading glasses made for the giant. In similar fashion, Jim has dentures and a wig made to solve the other problems.

We have used this book with fifth-grade children to consider proportional reasoning by focusing on multiplicative relationships. Suppose the giant is 9 feet (108 inches) tall, and that one of the children, Lynne, is 51 inches tall. We have asked the children a question similar to the following: "The giant is how many times as tall as Lynne?" That is, "How many Lynnes would need to be stacked on top of each other to equal the height of the giant?"

Children have approached this problem in several ways. Initially, some consider additive strategies, specifically adding Lynne's height until they reach the giant's height, or come fairly close to the giant's height. Others quickly consider multiplicative strategies by multiplying Lynne's height by successive numbers until they obtain a value as close as possible to the giant's height. In this case, they realize that $51 \times 2 = 102$ inches is too small but that $51 \times 3 = 153$ inches is too large. Successive iterations with multipliers between 2 and 3 permit the children to come fairly close to the giant's height. Typically, in considering how to determine the exact number of Lynnes needed to equal the giant's height, some child considers dividing the heights (i.e., $108 \div 51$). Thus, the giant is about 2.12 times Lynne's height.

Helping children at this level come to grips with multiplicative reasoning is not trivial. We have found that many children struggle with the idea of "A is how many times as big as B?" But by translating the question to "How many A's would need to be stacked on top of each other or laid end to end to equal B?", children are able to engage in this process of multiplicative reasoning. Further, in this process of reasoning, with its associated computation, they can also explore other mathematical concepts, such as measurement.

At this stage, we typically have children measure some common object that they regularly use, such as a toothbrush or a comb. They determine their height compared to the length of the toothbrush or the comb—namely,

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how many toothbrushes or combs would need to be stacked in order to equal their height. Using this value, they determine the appropriate length for the giant's toothbrush or comb in order for him to have the same comparison with his toothbrush that they have with theirs. Although ratios and proportions are not used formally in solving these problems, they are the underlying concepts inherent in the lesson. It is hoped that the informal work done with multiplicative relationships here will provide a foundation on which more-formal work with ratios and proportions can build in the middle grades.

The same multiplicative ideas help children at this level make scale drawings. We have had children locate an object (animal, plant, or building) to which they want to compare themselves. The children draw a picture of themselves 1 inch tall and a picture of the other object in the proper ratio.

For instance, one child chose to compare herself to a sun jellyfish. The book *Biggest, Strongest, Fastest* (Jenkins 1995) states that a sun jellyfish may be 200 feet long as it drags its tentacles through the water. Because the jellyfish is roughly 39 times as tall (long) as she is, a drawing of the jellyfish needs to be 39 inches if she is drawn 1 inch tall. On the classroom floor, she taped multiple pieces of paper together in order to construct her drawing. In a similar manner, other children have compared themselves to large objects.

Throughout these activities, the children were engaged in mathematics beyond what they were studying at that point in their regular mathematics curriculum. Such informal work with multiplicative reasoning has the potential to develop the type of fluency with numbers that we want children to have as they move into the middle grades.

PROPORTIONAL REASONING IN THE MIDDLE GRADES

At the middle grades, we have used a modified version of the Jonathan Swift classic about Gulliver, *Gulliver in Lilliput* (Hodges 1995), to engage middle grades students in activities focusing on proportional thinking. The modified version tells only the tale of Gulliver in the land of the Lilliputians. A number of children have seen all or part of the television miniseries about Gulliver, so that mathematics lessons based on this tale have a somewhat familiar setting. (An Internet site contains an edited version of *Gulliver's Travels* – www.jaffebros.com/lee/gulliver/bk1.)

The Lilliputians, who are only 6 inches tall compared to Gulliver's height of 6 feet (our choice for his height), make clothes for Gulliver by taking just two measurements, specifically the distance around the thumb and the length from his neck to the middle of his leg. The Lilliputians have found

that twice around the thumb is the distance around the wrist, twice around the wrist is the distance around the neck, and twice around the neck is the distance around the waist. When asked why the Lilliputians would only want to take a single girth measurement, middle grades students recognize that such small people would have a great deal of difficulty taking the measurements of such a large person, relatively speaking. That is, they seem to recognize that the proportional difference between Gulliver and the Lilliputians necessitates some considerations other than direct measurement.

We have had middle grades students, from grades 6–8, measure each of the lengths (thumb, wrist, neck, and waist) and record the results on the board, recording the measures separately for males and females. Table 14.1 contains data from one class of seventh-grade students. Although most students had no difficulty recording measurements on the board, even waist measurements, teachers should be sensitive to those students who are hesitant to include waist measurements, especially given that adolescents are often quite weight-conscious. Observe that for many of the students, the 2:1 ratios of the Lilliputians are in the ballpark, and for other students, the 2:1 ratio would yield rather inaccurate results. Hence, students can consider whether or not a single measure of their thumb, along with a length measurement, would permit a tailor to make a jacket with a reasonable chance of fitting.

TABLE 14.1.
Measures (in cm) from seventh-grade students to check the Lilliputian rule of thumb

Female				Male			
Thumb	Wrist	Neck	Waist	Thumb	Wrist	Neck	Waist
7.5	15	31	60	7	14.2	28.5	57.2
7.5	16	34	78	7	14	29	70
6.5	15	33	69	7	14	32.5	70
8	14	30	65	8	13	28	57
6	14	34	67	7	15	28	60
7.5	14	30.5	60	7.5	13.8	28	59
7.5	15	30	62.5				

In addition to taking measurements and comparing to the stated 2:1 ratio, students have graphed results, such as thumb-versus-neck measure. Notice that such a graph is roughly linear and can be described by the equation

$$\text{neck distance} = 4 \cdot \text{thumb distance}.$$

Likewise, students can determine the ratios that exist between waist and thumb or between waist and wrist.

As mentioned earlier, in the modified story the Lilliputians are 6 inches tall and Gulliver is 6 feet tall. Hence, the corresponding scale is 1 to 12. We have typically had students make a scale drawing of themselves, using a 1 to 10 or 1 to 20 scale so that their drawing fits on a regular sheet of paper. Although many students determine their scale drawing height by simply dividing their height by 10 or 20, respectively, the underlying mathematics involves proportional reasoning to realize that each 10 centimeters of their height is drawn as only 1 centimeter on paper.

In constructing their scale drawing, many focus only on their height in the scale drawing. Nevertheless, they make a drawing of themselves with reasonable proportions in terms of head, neck, legs, and so on. Other students have actually measured the length of their head, their neck, or their legs to guarantee that their scale drawing shows their body in the same proportion as in real life.

Another book that permits interesting comparisons for middle grades students is *Biggest, Strongest, Fastest* (Jenkins 1995). The book contains interesting facts about animals that set a record in their respective category. For instance, the best jumper for its size is the flea. Although a flea is only 1/16 inch tall, it can jump 8 inches into the air. Students can determine the ratio of the height jumped by the flea to its body height. How high could they jump if their jump height to body height were in the same ratio? If a typical story of a building is 10 feet, students are immensely impressed with themselves to think they could jump 65 to 70 stories.

We have found middle grades students to become quite engaged in doing mathematics as a result of reading one of these books. Not only do they like the stories, but also the contexts seem to offer insight into ways to engage in the mathematics. That is, the contexts make the mathematics less abstract and more concrete.

Literature provides an effective vehicle for engaging in mathematics, not only at the elementary grades but at the middle grades as well. Indeed, the mathematics embedded in many books connects well with the mathematics curriculum of grades 6–8. We encourage teachers to search actively for literature that can be used to form the basis of mathematics lessons or enhance the study of important concepts.

REFERENCES

Austin, Richard A., and Denisse R. Thompson. "Exploring Algebraic Patterns through Literature." *Mathematics Teaching in the Middle School* 2 (February 1997): 274–81.

Briggs, Raymond. *Jim and the Beanstalk*. New York: Putnam & Grosset, 1970.

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Hodges, Margaret. *Gulliver in Lilliput.* New York: Holiday House, 1995.

Jenkins, Steve. *Biggest, Strongest, Fastest.* New York: Ticknor & Fields Books, 1995.

Lawrence, Ann. "From *The Giver* to *The Twenty-One Balloons*: Explorations with Probability." *Mathematics Teaching in the Middle School 4* (May 1999): 504–09.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

Whitin, David J., and Sandra Wilde. *Read Any Good Math Lately? Children's Books for Mathematical Learning, K–6.* Portsmouth, N.H.: Heinemann, 1992.

———. *It's the Story That Counts: More Children's Books for Mathematical Learning, K–6.* Portsmouth, N.H.: Heinemann, 1995.

ANNOTATED BIBLIOGRAPHY

Ash, Russell. *Incredible Comparisons.* London: Dorling Kindersley, 1996.

This book provides comparisons for many different categories, such as the surface of the earth, going into space, great lengths, and animal speeds. Students can compare themselves to an appropriate benchmark or compare various objects in a given category. A wide range of ratios and proportions can be constructed from the facts in the book.

Beneduce, Ann Keay. *Gulliver's Adventures in Lilliput.* New York: Putnam & Grosset, 1996.

Gulliver lands in Lilliput, where all the people are only six inches tall. This is a slightly different version of the story from the one mentioned in the narrative of this chapter. Similar activities as those described in the chapter can be explored.

Briggs, Raymond. *Jim and the Beanstalk.* New York: Putnam & Grossett, 1970.

Jim meets the giant, Jack, and takes measurements of his head, his eyes, and his mouth in order to have a wig, a pair of glasses, and a pair of dentures made for the giant. Students can find corresponding measurements for classmates and determine a wide range of ratios similar to those in the chapter.

Carle, Eric. *The Grouchy Ladybug.* New York: Harper Trophy, 1977.

A ladybug flies from animal to animal, always wanting to start a fight. Even though the animals get larger as the ladybug's journey progresses, the ladybug always thinks the animal is too small to fight. Students can determine the number of ladybugs needed to measure a given object, using the length of a ladybug as the basis for the comparisons.

Clement, Rod. *Counting on Frank.* Milwaukee: Gareth Stevens Publishing, 1991.

The boy studies sizes and all types of facts. For instance, he determines how many of his dog would fit in a room, how long it would take to fill a bathtub, or how tall he would be if he grew at a given rate. The various rates mentioned in the book provide a context for exploring proportional thinking.

Hodges, Margaret. *Gulliver in Lilliput.* New York: Holiday House, 1995.

This book is a modified version of the Jonathan Swift classic, *Gulliver's Travels.* Activities similar to those in the chapter can be explored using the information about Gulliver's stay in Lilliput.

Jenkins, Steve. *Biggest, Strongest, Fastest.* New York: Ticknor & Fields Books, 1995.

This book presents information about a number of animals that are record holders in some way. Comparative information is also provided from which a variety of ratios and proportion problems can be developed.

Kellogg, Steven. *Much Bigger than Martin.* New York: Dial Books, 1976.

A young boy always feels too small when compared to his brother. He imagines what it would be like to be considerably bigger than his brother. Further, the book discusses some activities he could do if he were very tall, as well as some activities that he would not be able to pursue. Students could investigate how the sizes of buildings and other objects would need to change to accommodate various large individuals. Students could also investigate world records of "giants" in our lifetime.

Lasky, Kathryn. *The Librarian Who Measured the Earth.* Boston: Little, Brown & Co., 1994.

This is the story of Eratosthenes and how he measured the circumference of the Earth. Students can use ratios to determine distances walked by the bematists and determine how proportions were used to estimate the size of the earth.

Lord, John Vernon. *The Giant Jam Sandwich.* Boston: Houghton Mifflin Co., 1972.

A town makes a giant jam sandwich to capture pesky wasps. Students can explore the amount of ingredients needed to make items that appear in the *Guinness Book of World Records.* Ratios and proportions in expanding or shrinking recipes can also be investigated.

Malam, John. *Highest, Longest, Deepest: A Fold-Out Guide to the World's Record Breakers.* London: Simon & Schuster, 1996.

This book gives many natural examples that are record breakers for being the longest, highest, or deepest in their category. As with other comparison books, students can construct comparisons that are of interest to them, using informal multiplicative reasoning or more-formal proportions.

Most, Bernard. *How Big Were the Dinosaurs?* San Diego: Voyager Books, 1994.

Twenty different dinosaurs are introduced. Their sizes are compared to modern-day objects, providing multiple opportunities for the study of ratios and proportions.

Myller, Rolf. *How Big Is a Foot?* New York: Dell Publishing, 1962.

An apprentice tries to make a bed for the queen. Until he measures with the same foot size as the king, he is not able to build a bed that is the proper size. Not only does the book illustrate the importance of standard versus nonstandard measures, but also the book lends itself to ratio comparisons when different nonstandard measures are used.

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Suyeoka, George, Robert B. Goodman, and Robert A. Spicer. *Issunboshi*. Aiea, Hawaii: Island Heritage Publishing, 1974.

A one-inch boy protects a princess and wins her heart and hand. Students can consider appropriate sizes for a boat, a sword, and so on for a person who is only one inch tall. How loud would someone that size need to speak in order for a regular-sized person to hear them?

Wells, Robert E. *Is a Blue Whale the Biggest Thing There Is?* Morton Grove, Ill.: Albert Whitman & Co., 1993.

This book starts with the size of a blue whale and makes comparisons with larger and larger objects as it considers the size of the universe. Relative sizes of the planets, distances between the planets, and travel at the speed of light are all possible topics based on this book. Students can explore the astronomical comparisons in which measures are based on an earth measure of 1.

———. *What's Faster than a Speeding Cheetah?* Morton Grove, Ill.: Albert Whitman & Co., 1997.

Speeds of different objects are compared, including animals, planes, spaceships, and so on. The various facts lend themselves to a variety of ratio and proportion comparisons.

———. *What's Smaller than a Pygmy Shrew?* Morton Grove, Ill.: Albert Whitman & Co., 1995.

This book uses a pygmy shrew to make comparisons of very small objects, focusing on the molecular level. Ratios and proportions with small objects are possible avenues of study, including the magnification ratios of microscopes.

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